

A preliminary analysis of the energy transfer between the dark sectors of the Universe

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Abstract

We study the mutual interaction between the dark sectors (dark matter and dark energy) of the Universe by resorting to the extended thermodynamics of irreversible processes and constrain the former with supernova type Ia data. As a byproduct, the present dark matter temperature results in good agreement with independent estimates of the temperature of the gas of sterile neutrinos.

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It is widely held by now that the accelerated expansion of the Universe can be traced to some invisible agent (possibly the quantum vacuum or some or other scalar field), dubbed dark energy (DE), which takes slightly over 70% of the energy budget and is endowed with a high negative pressure -see [1] for reviews. The possibility that this agent interacts with the remaining fields of the standard model is more natural and general than otherwise whence it is receiving growing attention -see [2] and references therein. However, local gravity experiments set severe bounds on the interaction with conventional matter (e.g., baryons) [3] but there is nothing against a possible interaction with dark matter (DM). Unless there exists an underlying symmetry that would set the interaction (coupling) to zero (such a symmetry is still to be discovered) there is no *a priori* reason to toss it away. Moreover, the existence of this coupling is undeniable on physical grounds: since DE energy gravitates it must be accreted by massive compact objects like black holes and neutron stars. In a cosmological context this energy transfer from DE to DM must be small but non-vanishing.

The coupling between DE and dark matter (DM), first introduced to lower down the huge value of the cosmological constant [4], can provide a mechanism to alleviate the coincidence problem [5, 6, 7] or even solve it [8]. Furthermore, it has been argued that an appropriate interaction between DE and DM can influence the perturbation dynamics and affect the lowest multipoles of the CMB spectrum [9, 10]. Recently, it has been shown that such a coupling could be inferred from the expansion history of the Universe, as manifested in the supernova data together with CMB and large-scale structure [11, 12, 13, 14]. Signatures of the interaction between DE and DM in the dynamics of galaxy clusters has also been analyzed [15, 16]. Further discussions on the interaction between dark sectors can be found in [17, 18].

The aforesaid interaction has also been considered from a thermodynamical perspective -see e.g. [19, 20]. Assuming that the DE can be treated as a fluid with a well defined temperature, it has been argued that if at present there exists a transfer of energy between DE and DM, it must be such that the latter gains energy from the former and not the other way around for the second law of thermodynamics [21] to be fulfilled. The authors of Ref. [20] considered a system composed of two subsystems (DM and DE) at different

temperatures. In virtue of the extensive property, the entropy of the whole system is the sum of the entropies of the individual subsystems which (being equilibrium entropies) are just functions of the energies of DE and DM even during the energy transfer process.

The aim of this Letter is to look more closely into the aforementioned transfer between both subsystems by resorting the thermodynamics of irreversible processes as formulated by Jou *et al.* [22]. There, to cope with nonequilibrium (i.e., irreversible) situations, the entropy is generalized by allowing nonequilibrium quantities to enter it. Recall that, by definition, nonequilibrium quantities vanish at equilibrium and, consequently, this generalized entropy reduces to the conventional equilibrium entropy in that limit.

If DE and DM conserve separately in an expanding Friedmann-Lemaître-Robertson-Walker universe we would have

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad (1)$$

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = 0, \quad (2)$$

where the equation of state of DM can be approximately written in parametric form as [23]

$$\rho_m = n_m M + \frac{3}{2} n_m T_m, \quad p_m = n_m T_m \quad (k_B = 1) \quad (3)$$

provided that $T_m \ll M$. Therefore, $\rho_m \sim a^{-3}$ and $\rho_x \propto \exp \int -3(1 + w_x) da/a$. Here, $w_x = p_x/\rho_x < 0$ is the equation of state parameter of dark energy.

The dependence of both temperatures on the scale factor

$$T_m \propto a^{-2}, \quad T_x \propto \exp \int -3w_x da/a \quad (4)$$

follows from integrating the evolution equation, $\dot{T}/T = -3H(\partial p/\partial \rho)_n$ for each subsystem. The latter is a consequence of Gibbs' equation, $TdS = d(\rho/n) + p d(1/n)$ and the condition for dS to be a differential expression. Since $w_x < 0$, equations (4) suggest that currently $T_m \ll T_x$.

When the DE and DM components interact with each other, Eqs. (1) and (2) generalize

to

$$\dot{\rho}_m + 3H(\rho_m + p_m) = \Gamma, \quad (5)$$

and

$$\dot{\rho}_x + 3H(1 + w_x)\rho_x = -\Gamma, \quad (6)$$

respectively, where Γ denotes the interaction term. For $\Gamma > 0$ the energy proceeds from dark energy to dark matter.

For simplicity we assume the specific coupling

$$\Gamma = 3H\lambda\rho_x, \quad (7)$$

with λ a small, dimensionless, positive quantity. This kind of models has been considered in the literature -see e.g. [14, 24, 25]- and show compatibility with observation [26].

Upon neglecting the DM pressure, the ratio between the energy densities, $r = \rho_m/\rho_x$, is seen to obey

$$\frac{dr}{dx} = 3(\lambda + \lambda r + w_x r), \quad (8)$$

where $x = \ln a$. When w_x and λ are constants the latter equation integrates to

$$r(x) = \frac{[r_{(eq)}(\lambda + w_x) + \lambda] e^{3(\lambda + w_x)(x - x_{(eq)})} - \lambda}{\lambda + w_x}, \quad (9)$$

The subscript (eq) indicates the value taken by the corresponding quantity when DE and DM are in thermal equilibrium.

In the scaling regime, i.e., when the ratio r stays constant, the coincidence of the two dark components is understood [6, 7]. Outside this regime r may be considered piecewise constant, especially for $z \leq 20$. In particular, $\lambda = -\frac{w_x r_0}{r_0 + 1}$ about the present time, where $r_0 \simeq 3/7$ is the current r value.

When the interaction is taken into account, temperatures of DE and DM evolve as

$$T_x = T_{(eq)} e^{-3(w_x + \lambda)(x - x_{(eq)})}, \quad \text{and} \quad T_m = T_{(eq)} \frac{r}{r_{(eq)}} e^{-[2 + 3(\lambda + w_x)](x - x_{(eq)})}, \quad (10)$$

respectively. It is immediately seen that both temperatures vary more slowly than in the noninteracting case (recall that $0 < \lambda < |w_x|$), see Fig.1.

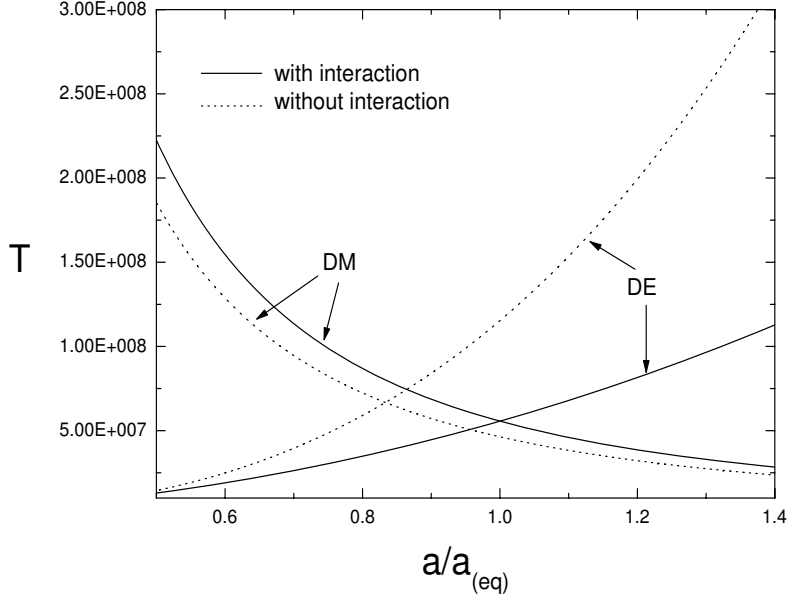


FIG. 1: Evolution of the temperatures (in Kelvin degrees) of DM and DE in the non-interacting case (dotted lines), and in the interacting case (solid lines). In drawing the figure we have taken $w_x = -1$ and $\lambda = 0.3$.

Consider an isolated system composed of two subsystems, one being DE and the other DM, satisfying

$$T_m \frac{dS_m}{dt} = \frac{dQ_m}{dt} = \frac{dE_m}{dt}, \quad \text{and} \quad T_x \frac{dS_x}{dt} = \frac{dQ_x}{dt} = \frac{dE_x}{dt} + p_x \frac{dV}{dt}, \quad (11)$$

respectively, with $E_m = \rho_m V$, $E_x = \rho_x V$ and $V = a^3$. Since the overall system is isolated, one has

$$\frac{dQ_m}{dt} = -\frac{dQ_x}{dt} = -\dot{Q}, \quad (12)$$

where $\dot{Q} = -a^3 \Gamma$ is the energy transfer rate. In the comoving volume, this is nothing but the energy conservation law: $\dot{\rho}_x + \dot{\rho}_m + 3H(\rho_m + \rho_x + p_x) = 0$.

In [20], the entropy of the whole system depends on the energy densities and volume only, and in virtue of the extensive property, it is just the sum of the entropies, S_m and S_x , of the subsystems. In equilibrium thermodynamics [21], irreversible fluxes -such as energy transfers- play no part and they do not enter the entropy function which is defined for equilibrium states only. However, in nonequilibrium extended thermodynamics such fluxes

enter the entropy function, S^* , which is more general than S as it can be defined also outside equilibrium [22]. In this spirit, we postulate that $S^* = S^*(\rho_m, \rho_x, V, \dot{Q})$. Accordingly, its time variation is given by

$$\frac{dS^*}{dt} = T_m^{-1} \frac{dQ_m}{dt} + T_x^{-1} \frac{dQ_x}{dt} - \tilde{\Gamma}(\dot{Q}) \frac{d\dot{Q}}{dt}, \quad (13)$$

where $-\tilde{\Gamma}(\dot{Q})$ is defined as the derivative of S^* with respect to \dot{Q} and we have used Eqs. (11). For the sake of simplicity, we suppose $\tilde{\Gamma}(\dot{Q}) = A\dot{Q}$, with A a semi-positive definite constant.

Assuming r piecewise constant, the energy transfer rate at present time can be roughly determined as

$$\left. \frac{d\dot{Q}}{dt} \right|_0 \simeq -\frac{27\lambda a_0^3 H_0^3}{8\pi(1+r_0)} (\dot{H} + H^2)_0 < 0, \quad (14)$$

since the Universe entered the accelerated regime only recently. Thus, in the current accelerating stage, the second law in extended irreversible thermodynamics, $dS^*/dt \geq 0$, implies

$$A(d\dot{Q}/dt) \geq T_x^{-1} - T_m^{-1}. \quad (15)$$

In view of the evolution of DE and DM temperatures, this inequality can be satisfied for any negative value of A . However, for positive values, there must be an upper bound on A to guarantee the second law.

In virtue of the expressions (14) and (10), Eq. (15) can be recast around the present time as

$$-A'H^3 \left(\frac{dH}{dx} + H \right) \geq e^{2.1x_{(eq)}} e^{-5.1x} - \frac{3}{7} r_{(eq)} \left\{ r_{(eq)} e^{2x_{(eq)}} e^x + e^{-0.1x_{(eq)}} e^{3.1x} \right\}^{-1}, \quad (16)$$

where $A' = 567 T_{(eq)} A / (800\pi)$ and we have set w_x and r_0 to -1 and $3/7$, respectively. Approximating the Hubble function by $H \simeq H_0 - H_0(1+q_0)x$, with q_0 the current value of the deceleration parameter, integration of the above inequality yields

$$H^4 \geq H_0^4 - \frac{4H_0^4}{5(1+q_0)} + \frac{4H_0^4}{5(1+q_0)} [1 - (1+q_0)x]^5 + \frac{4e^{2.1x_{(eq)}}}{5.1 A'} (e^{-5.1x} - 1) - \frac{4}{A'} \int_x^0 \tilde{f}(x', r^{(eq)}, x_{(eq)}) dx', \quad (17)$$

where $\tilde{f}(x', r_{(eq)}, x_{(eq)}) = r_{(eq)} \left\{ (r_{(eq)} - \frac{3}{7}) e^{2x_{(eq)}} e^x + \frac{3}{7} e^{-0.1x_{(eq)}} e^{3.1x} \right\}^{-1}$. Using $x = -\ln(1+z)$

we rewrite the inequality as

$$H^4(z) \geq H_0^4 - \frac{4H_0^4}{5(1+q_0)} + \frac{4H_0^4}{5(1+q_0)}[1 + (1+q_0) \ln(1+z)]^5 \\ + \frac{4(1+z_{(eq)})^{-2.1}}{5.1 A'} [(1+z)^{5.1} - 1] - \frac{4}{A'} \int_0^z \frac{\tilde{g}(z', z_{(eq)}, r^{(eq)})}{1+z'} dz', \quad (18)$$

where $\tilde{g}(z, z_{(eq)}, r_{(eq)}) = r_{(eq)} \left\{ [r_{(eq)} - \frac{3}{7}] (1+z_{(eq)})^{-2} (1+z)^{-1} + \frac{3}{7} (1+z_{(eq)})^{0.1} (1+z)^{-3.1} \right\}^{-1}$.

With the help of the expressions for the luminosity distance and distance modulus,

$$d_L = c(1+z) \int_0^z \frac{dz}{H(z)}, \quad \text{and} \quad \mu(z) = 5 \log(d_L) + 25,$$

one obtains the following inequality for the latter

$$\mu(z) \leq 5 \log \left[c(1+z) \int_0^z h(z', A', z_{eq}, r_{(eq)})^{-\frac{1}{4}} dz' \right] + 25, \quad (19)$$

where c is the speed of light and $h(z, A', z_{(eq)}, r_{(eq)})$ stands for the right-hand-side of Eq. (18).

The independent parameters $r_{(eq)}$, $z_{(eq)}$, and A' can be roughly appraised by resorting the second law and SN Ia data. Here we shall use the sample of the recent golden SNe Ia, compiled in [27], in a similar way as in [28]. Since we have employed a linear approximation in the above integration and focused on the present accelerated era, we will use only low redshift data, namely $z \leq 0.5$.

Taking the equality sign in (19), setting $H_0 = 72$ Km/s/Mpc, and fitting the distance modulus to the supernovae data, the best fit values of the parameters are found to be $A' = 6.42 \times 10^{-3}$, $r_{(eq)} = 1.09 \times 10^5$ and $z_{(eq)} = 5.56 \times 10^7$, with $\chi^2 = 73.6178102$. We remark that $z_{(eq)}$ has only a small influence on the total χ^2 value when latter lies in the range $10^3 \leq z_{(eq)} \leq 10^8$. The value for A' quoted above is the upper bound set by the second law. For values exceeding that one the distance modulus curve (depicted in Fig.2) would run below the SNe Ia data, i.e., the second law, $dS^*/dt \geq 0$, would be violated in the region above the curve.

As reasoned above (see also [20]), after the thermal equilibrium between DM and DE is lost (i.e., for $z < z_{(eq)}$), a transfer of energy between these sectors will arise and T_x will increase more slowly than in the absence of interaction and, correspondingly, T_m will

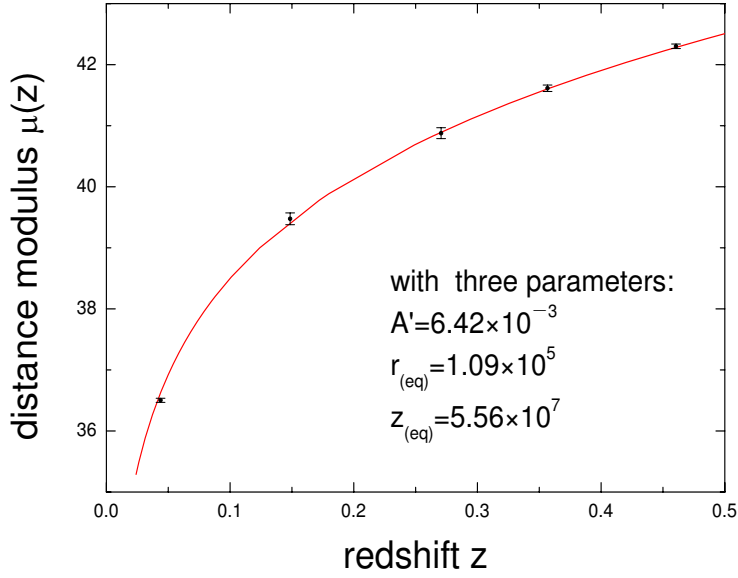


FIG. 2: The graph sets the border above which the second law of thermodynamics would be violated.

decrease more slowly -see Fig. 1. This does not bring the systems to any new equilibrium, but it certainly slows down the rate at which they move away from mutual equilibrium. To assess quantitatively the relevance of the interaction appearing in the expression for dS^*/dt we compute the current value of the ratio between the third and first terms in Eq. (13), i.e.,

$$R_{m0} = \frac{|A\dot{Q}(d\dot{Q}/dt)|_0}{|T_m^{-1}(dQ_m/dt)|_0} = \frac{11\alpha}{20r_{(eq)}} \left\{ (r_{(eq)} - \frac{3}{7}e^{2x_{(eq)}} + \frac{3}{7}e^{-0.1x_{(eq)}}) \right\}, \quad (20)$$

where $\alpha = A' H_0^4$. Inserting the best fitting values for A' , $r_{(eq)}$ and $z_{(eq)}$, we get $R_{m0} \simeq 1.4$. Thus, the term coming from the interaction should not be neglected in the expression for the entropy variation.

We next compare the present DM temperature with and without coupling between the dark sectors. In the absence of interaction, in virtue of Eq. (4), we would have

$$T_{m0} = T_{(eq)}(1 + z_{eq})^{-2} \simeq \frac{T_{(eq)}}{z_{(eq)}^2}, \quad (21)$$

and the thermal equilibrium between DE and DM should have occurred earlier (at larger $z_{(eq)}$) than in the interacting case. If we consider $T \sim z$ and use the best fitting value of $z_{(eq)}$,

we infer that $T_{m_0} \sim 10^{-7}$ Kelvin. In the noninteracting case, the current DM temperature can be even smaller. In the interacting case, the present temperature of DM can be estimated as

$$T_{m_0} = \frac{r_0}{r_{(eq)}} T_{eq} (1 + z_{(eq)})^{0.1}. \quad (22)$$

As noted above $z_{(eq)}$ cannot be strictly constrained since χ^2 shifts only by a factor of about 10^{-5} when $z_{(eq)}$ varies from 10^3 to 10^8 . For $(\alpha, r_{(eq)}, z_{eq}) = (1.64 \times 10^5, 0.8 \times 10^5, 3.87 \times 10^4)$ — what implies $\chi^2 = 73.61782324$, and we obtain $T_{m_0} \simeq 0.596$ Kelvin. This value is consistent with the estimations of Hansen *et al.* [29] based on simulations of the number of satellite galaxies and astronomical upper limit of the sterile neutrinos -one of the best promising DM candidates- with temperature

$$T_{w_0} = T_{\nu_0} \left(\Omega_w h^2 \frac{94 \text{ eV}}{m_w} \right)^{1/3} \simeq 0.581 \text{ Kelvin} \quad (23)$$

(see Eq. (1) in Ref. [29]), where $T_{\nu_0} = 1.946$ Kelvin, $\Omega_w = 0.3$, $h = 0.72$ and the lower bound $m_w = 550$ eV [30].

In summary, we have further employed the second law of thermodynamics to study the coupling between the dark sectors of the Universe. More specifically, by using the nonequilibrium entropy, S^* , of extended irreversible thermodynamics alongside the gold SNe Ia data of Riess *et al.* [27] we roughly determined the redshift, $z_{(eq)}$, at which DE and DM were momentarily in thermal equilibrium as well as the ratio between their energy densities at that moment. In addition, we estimated an upper bound on the phenomenological parameter A' . We would like to emphasize that in the absence of any mutual interaction between dark sectors the DM temperature results extremely low. However, due to the interaction it can meet the independent astronomical upper bound on warm DM particles [29].

Admittedly, our analysis is just preliminary and presents some limitations. In the first place, we did not specify microscopically the process by which the two dark sectors couple with one another -this lies far beyond the scope of this work. We only proposed a phenomenological expression for the interaction (cf. Eq. (7)). Secondly, we expanded $\tilde{\Gamma}(\dot{Q})$ just to first order in \dot{Q} . A more ambitious treatment should go to higher orders. However, this would introduce additional unknown parameters and complicate the numerical

analysis, something to be concerned with in view of the scarcity and limitations of current observational data.

This said, we believe our study may well open the way to fuller investigations of the DE/DM interaction in the light of nonequilibrium thermodynamics.

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